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RZ-003-1016003 Seat No. _____

Third Year B. Sc. (Sem. VI) (CBCS) Examination

March - 2019

Mathematics : Paper - 10 (A)

(Optimization and Numerical Analysis - II)

(Theory) (New Course)

Faculty Code : 003

Subject Code : 1016003

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.

(2) Numbers written to the right indicate full marks of the question.

1 (A) Attempt the following : **1+1+1+1=4**

(1) Define : Slack Variables (with respect to Linear Programming Problems)

(2) Define : Basic Feasible Solution (with respect to Linear Programming Problems).

(3) Define : Convex Linear Combination.

(4) Define : Strictly Convex Function.

(B) Attempt any **one** in brief : **2**

(1) Define :

(i) A non-degenerate B.F.S.

(ii) Surplus Variables with respect to Linear Programming Problems

(2) Write the Matrix Form of Linear Programming Problem.

(C) Attempt any **one** in detail : **3**

(1) Read the following description of the problem, use the same to FORMULATE (ONLY) the problem into mathematical form of Linear Programming Problem (NO need to obtain the solution)

Description of Linear Programming Problem :

"A firm can produce three types of cloths say A, B and C. Three kinds of wool are required for it, say red wool, green wool and blue wool. One unit length of type A cloth needs 2 meters of red wool and 3 meters of blue wool. One unit length of type B cloth needs 3 meters of red wool, 2 meters of green wool and 2 meters of blue wool. One unit length of type C cloth needs 5 meters of green wool, and 4 meters of blue wool. The firm has only a stock of 8 meters of red wool 10 meters of green wool and 15 meters of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3.00 of type B cloth is Rs. 5.00 and of type C cloth is Rs. 4.00 Determine how the firm should use the available material so as to maximize the income from the finished cloth. "

(2) Explain the steps of the Graphical Method to solve, the Linear Programming Problems.

(D) Attempt any **one** : **5**

(1) Explain steps of Big M Method to solve the Linear Programming Problems.

(2) Use TWO PHASE method to find the ONLY initial B.F.S. of the following linear programming problem (complete the PHASE - I only, NO need to obtain FINAL solution)

Minimize $Z = x_1 + x_2$

Subject to the constraints

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

and $x_1, x_2 \geq 0$.

2 (A) Attempt the following : **1+1+1+1=4**

(1) What is the full form of NWCM ?

(2) What is the name of the method to find optimum solution of transportation problem?

(3) If there is a constraint of = type (an equation as a constraint) in a primal linear programming problem then the dual of the same will have a corresponding variable which is unrestricted in sign. True or False?

(4) Out of two methods NWCM and LCM (to find initial solution of transportation problem) which method is a better method? Why?

(B) Attempt any **one** in brief : **2**

(1) State the general mathematical form of Assignment Problem.

(2) Solve the following assignment problem

		Subordinates			
		I	*II*	*III*	*IV*
Tasks	*A*	8	26	17	11
	B	13	28	4	26
	C	38	19	18	15
	D	19	26	24	10

(C) Attempt any **one** in detail : **3**

(1) Obtain the INITIAL solution of given transportation problem using LCM method.

		To				
		D_1	D_2	D_3	D_4	*Supply*
	S_1	19	30	50	10	7
From	S_2	70	30	40	60	9
	S_3	40	8	70	20	18
	Demand	5	8	7	14	34

(2) Obtain the dual of the following primal Linear Programming Problem :

Minimize $Z = 5x_1 + x_2 - 6x_3$

Subject to the constraints

$-2x_1 + x_2 + 11x_3 \leq -2$

$-x_1 + 7x_2 + x_3 \geq 7$

$3x_1 - x_2 + 4x_3 \leq 5$

and $x_1, x_2, x_3 \geq 3.$

(D) Attempt any **one** : **5**

(1) Write the steps of Lowest Cost Entry Method to find initial solution of transportation problem.

(2) Obtain the OPTIMUM solution of the following Transportation Problem using MODI method.

		Destination				
		D_1	D_2	D_3	D_4	*Supply*
	S_1	5	3	6	4	30
Origin	S_2	3	4	7	8	15
	S_3	9	6	5	8	15
	Demand	10	25	18	7	60

3 (A) Attempt the following : **1+1+1+1=4**

(1) The operator $\delta =$ _____Δ (fill in the blank)

(2) Define : interpolation

(3) What is special case of Bessel's formula?

(4) What is the third divided difference of the polynomial of second degree?

(B) Attempt any **one** in brief : **2**

(1) If $f(x) = x^3$ then find $f(2,4,6)$.

(2) What is the drawback of Lagrange's interpolation formula?

(C) Attempt any **one** in detail : **3**

(1) Use Lagrange's formula to find the value of y at $x = 6$ from the following data.

x	5	6	9	11
y	12	13	14	16

(2) If $y_2 = 10, y_1 = 8, y_0 = 5, y_{-1} = 10$ then using Gauss forward interpolation formula, find the value of $y_{\frac{1}{2}}$.

(D) Attempt any **one** : **5**

(1) State and prove Gauss Forward Interpolation Formula.

(2) Show that n^{th} divided difference of polynomial of n^{th} degree is constants.

4 (A) Attempt the following : **1+1+1+1=4**

(1) What is numerical integration?

(2) Write General quadrature formula.

(3) Write trapezoidal rule.

(4) General quadrature formula is also known as _____

(B) Attempt any **one** in brief : **2**

(1) In usual notation prove that

$$D^3 = \frac{1}{h^3}\left[\nabla^3 + \frac{3}{2}\nabla^4 + \frac{7}{4}\nabla^5 + \ldots.\right]$$

(2) Find the value of $\int_2^6 \frac{dx}{x}$ by trapezoidal rule.

(C) Attempt any **one** in detail : **3**

(1) Derive Trapezoidal rule.

(2) Evaluate $\int_2^6 \frac{dx}{x}$ by Simpson $\frac{1}{3}$ rule

(D) Attempt any **one** : **5**

(1) Derive Simpson's $\frac{1}{3}$ rule.

(2) Derive Simpson's $\frac{3}{8}$ rule

5 (A) Attempt the following : **1+1+1+1=4**

(1) Write Taylor formula to solve ordinary differential equation.

(2) Write Picard's formula to solve ordinary differential equation.

(3) Write Range's formula to solve ordinary differential equation.

(4) Write Range Kutta's formula to solve ordinary differential equation.

(B) Attempt any **one** in brief : **2**

(1) Using Picard's formula to find $y(0.1)$ given that $\frac{dy}{dx} = x + y,\ y(0) = 1,\ h = 0.1$

(2) Find the value of y at $x = 0.2$ by Taylor's method $y' = 2y + 3e^x, y(0) = 0.$

(C) Attempt any **one** in detail : **3**

(1) Explain Euler's method to solve ordinary differential equation.

(2) Find the value of y at $x = 0.2, 0.4$ by Euler's method $\frac{dy}{dx} = 2x + y,\ y(0) = 1$

(D) Attempt any **one** : **5**

(1) Explain Taylor methods to solve ordinary differential equation.

(2) Explain Range Kutta's formula to solve ordinary differential equation.
